CANT 2017 Abstracts

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Ali Armandnejad, Vali-e-Asr University of Rafsanjan, Iran
Title: Number of signature matrices on each component of J-orthogonal matrices
Abstract: Let $\mathbf{M}_n$ be the set of all $n \times n$ real matrices. A matrix $J \in \mathbf{M}_n$ is said to be a signature matrix if J is diagonal and its diagonal entries are ± 1. A matrix $A \in \mathbf{M}_n$ is said to be a J-orthogonal matrix if there exists a signature matrix J such that $A^\top JA = J$. It is a well known topological result that for $J \neq \pm I$, the set of all $n \times n$ J-orthogonal matrices has four connected components. In this note we find the number of signature matrices that is placed on each component of the set of $n \times n$ J-orthogonal matrices.

Abdul Basit, Rutgers - New Brunswick
Title: Recent improvements in sum-product estimates
Abstract: The sum-product conjecture of Erdős-Szemerédi states that for any finite subset A of the reals $\max\{|A+A|, |AA|\} \geq |A|^{2-\epsilon}$. Here $A+A$ (resp. AA) is the set of all pairwise sums (resp. products) of elements of A. We will discuss some recent improvements for various sum-product type estimates.

Sam Chow, University of York, England
Title: Bohr sets and multiplicative diophantine approximation
Abstract. In two dimensions, Gallagher's theorem is a strengthening of the Littlewood conjecture that holds for almost all pairs of real numbers. We prove an inhomogeneous fibre version of Gallagher's theorem, sharpening and making unconditional a result recently obtained conditionally by Beresnevich, Haynes and Velani. The idea is to find large generalised arithmetic progressions within inhomogeneous Bohr sets, extending a construction given by Tao. This precise structure enables us to verify the hypotheses of the Duffin–Schaeffer theorem for the problem at hand, via the geometry of numbers.

David Chudnovsky and Gregory Chudnovsky, NYU
Title: Near isomorphism of graphs

Colin Defant, University of Florida
Title: A new proof of Euler's formula for $\zeta(2k)$
Abstract: Let $f_k\colon \mathbb{R} \to \mathbb{R}$ be the function that is periodic with period 2π and satisfies $f_k(x) = x^k$ whenever $-\pi < x \leq \pi$. Analyzing the Fourier coefficients of these functions yields a quick inductive proof that $\zeta(2k)$ is a rational multiple of π^{2k} for every positive integer k. By attending to details, we provide a new inductive proof of Euler's classical formula

$$\zeta(2k) = \frac{(-1)^{k+1} B_{2k} (2\pi)^{2k}}{2(2k)!}.$$

Along the way, we also find a new identity involving Bernoulli numbers.
This is joint work with K. Alladi.

Robert Donley, Queensborough Community College (CUNY)
Title: Clebsch-Gordan coefficients in characteristic zero
Abstract: Clebsch-Gordan coefficients for SL(2, C) play a central role in representation theory, classical particle physics, and special functions. Known closed formulas (Wigner, Racah, Shapiro) involve an alternating sum, factorials, and square roots. By adjusting the usual conventions, we obtain a closed formula with rational values, an associated generating function, and an efficient algorithm for computation by hand.

Joseph Gunther, CUNY Graduate Center
Title: Slicing the stars: Counting algebraic numbers by degree and height"
Abstract: Masser and Vaaler have given an asymptotic formula for the number of algebraic numbers of given degree d and increasing height. This problem was solved by counting lattice points (which correspond to minimal polynomials over $\mathbb{Z}$) in a homogeneously expanding star body in $\mathbb{R}^{d+1}$. The volume of this star body was computed by Chern and Vaaler, who also computed the volume of the codimension-one "slice" corresponding to monic polynomials – this led to results of Barroero on counting algebraic integers. We'll discuss how to estimate the volume of higher-codimension slices, which allows us to count units, algebraic integers of given norm, trace, norm and trace, and more.
This is joint work with Robert Grizzard.

Brandon Hanson, Pennsylvania State University
Title: Integers not represented by diagonal forms of large degree
Abstract: Waring's Problem tells us that for fixed k and large enough s, every number can be written can be written as a sum of s perfect kth powers. If, instead, we fix s and let k grow, such a result is blatantly false - to represent N, we are limited to the kth powers bounded by $N^{1/k}$, and there just aren't enough kth powers to go around. On the other hand, if we work with sums and differences, this fact is no longer present (we are now representing numbers with an indefinite form). None the less, we still expect that the number of integers represented to be quite small when k is large relative to s. Such a statement is only known to be true conditional on the generalized *abc*-conjecture. In joint work with Asif Zaman, we aim for a more modest, though unconditional, result: The numbers represented by a diagonal form of large degree are usually not too dense.

Charles Helou, Pennsylvania State University - Brandywine
Title: Cyclotomic expressions for representation functions
Abstract: Given a subset A of the set $\mathbb{N} = \{0, 1, 2, \cdots\}$ of natural numbers or of the residue class ring $\mathbb{Z}/N\mathbb{Z}$, where N is a fixed positive integer, we introduce certain exponential sums associated with A. They allow us to obtain expressions for the classical functions that characterize A, in particular the additive representation function of A. The expressions are first given directly in terms of the exponential sums, but then also in terms of algebraic forms such as the trace forms in subfields of a cyclotomic field. These sums are also of intrinsic interest, and we note some of their properties.

Brian Hopkins, St. Peter's University
Title: Magic configurations
Abstract: An (n_k) configuration is a set of n points and n lines such that every point lies on k lines and every line contains k points. These geometric/combinatorial objects date back to Pappus and saw a flurry of attention in the late 19th century but then suffered a lull until a 1990 renaissance due to Branko Grünbaum. Here we consider a question motivated by magic squares and the like: Can the points of a configuration be labeled with distinct integers such that the sums for each line are the same? We restrict our attention to $k = 3$. Results include two approaches to finding the minimal n for which a magic (n_3) configuration exists and a construction showing that there are infinitely many such configurations.
This is joint work with Michael Raney of Georgetown University.

Robert Hough, SUNY at Stony Brook
Title: The shape of cubic fields
Abstract: The ring of integers of a cubic field K is a lattice in the canonical embedding of K in $\mathbb{R}^3$ or $\mathbb{R} \times \mathbb{C}$. I will discuss ongoing work which studies this shape spectrally when cubic forms are ordered by increasing size of the discriminant. The method builds on the ζ-function approach of Shintani and Taniguchi-Thorne.

Alex Iosevich, University of Rochester
Title: On a two-parameter variant of the Erdős distance problem
Abstract: Given $E \subset \mathbf{R}^2$, let $B_{2,2}(E) = \{(|x' - y'|, |x'' - y''|) : x = (x', x''), y = (y', y'') \in E\}$. The question we ask is, if $\#E = n$, how small can $\#B_{2,2}(E)$ possibly be?

William J. Keith, Michigan Technological University
Title: Major index over descent for pattern avoidance classes: unimodality and other combinatorial features
Abstract: The Eulerian-Mahonian bivariate generating function for the joint distribution of the major index and descent statistics of permutations of length n is a classical object with many well-established features. Less studied but perhaps equally interesting is the same generating function with the summation restricted to pattern avoidance classes. We shall discuss the usual questions of symmetricity, unimodality, and recurrences, as well as some observations of interesting combinatorial features to motivate future research.

Mizan R. Khan, Eastern Connecticut State University
Title: An *amuse-bouche* arising from a result of Mordell
Abstract: Let p be a prime with $p \equiv 3 \pmod 4$, and consider the modular hyperbola

$$\mathcal{H}_p = \{(x, y) : xy \equiv 1 \pmod p, 1 \leq x, y \leq p - 1\}.$$

We have that

$$\left(\frac{p-1}{2}\right)! \equiv (-1)^a \pmod p.$$

Mordell, in the February 1961 issue of the *Amer. Math. Monthly*, gave an elegant criterion to determine the parity of a. We apply this criterion to determine the parity of elements of $\mathcal{H}_p$ that lie in the triangle with vertices $(0, 0), (0, p/2), (p/2, p/2)$.

Byungchan Kim, SeoulTech, Republic of Korea
Title: (q, t)-Delannoy numbers and integer partitions
Abstract: We introduce an overpartition analogue of Gaussian polynomials $\overline{\begin{bmatrix} m+n \\ n \end{bmatrix}}_{q,t}$ as the generating function for overpartitions fitting inside an $m \times n$ rectangle, which is also a (q, t)-analogue of Delannoy numbers. First we obtain finite versions of classical q-series identities such as the q-binomial theorem and the Lebesgue identity, as well as two-variable generalizations of classical identities involving Gaussian polynomials. Then, by constructing involutions, we obtain an identity involving a finite theta function and prove the (q, t)-log concavity of $\overline{\begin{bmatrix} m+n \\ n \end{bmatrix}}_{q,t}$. We particularly emphasize the role of combinatorial proofs and the consequences of our results on Delannoy numbers.
This is joint work with Jehanne Dousse.

Hershy Kisilevsky, Concordia University, Canada
Title: Sign matrices and quadratic residue matrices
Abstract: Sign matrices are $n \times n$ matrices with diagonal entries equal to zero and off diagonal entries ± 1. A sign matrix is a Quadratic Residue (QR) matrix if the entries are Legendre Symbols. QR matrices arise in the study of the decomposition of the ramified primes in tamely minimally ramified extensions $K/\mathbf{Q}$ which are composites of quadratic extensions. We characterize QR matrices among the $n \times n$ sign matrices and show that the proportion of QR matrices becomes vanishingly small as $n \to \infty$.
This is joint work with D.S. Dummit and E. Dummit.

Sándor Kiss, Budapest University of Technology and Economics, Hungary
Title: On the structure of sets which has coincide representation functions
Abstract: For a set of nonnegative integers S let $R_S(n)$ denote the number of unordered representations of the integer n as the sum of two different terms from S. Together with Csaba Sándor we partially described the structure of the sets, which has coincide representation functions by using Hilbert cubes.

Nana Li, Bard College at Simon's Rock
Title: On the union-closed sets conjecture
Abstract: A family of finite sets is called *union-closed* if it contains the union of any two sets in it. The *Union-Closed Sets Conjecture* of Frankl from 1979 states that each union-closed family contains an element that belongs to at least half of the members of the family. In this paper, we will talk about the recent development of this conjecture and several other related conjectures.

Jared Lichtman, Dartmouth College
Title: Lying on the Fermat primality test
Abstract: We investigate the probability that a random odd composite number passes a random Fermat primality test, improving on earlier estimates in moderate ranges. For example, with random numbers to 2^200, our results improve on prior estimates by close to 3 orders of magnitude. We also provide the true probabilities in smaller ranges. These results, combined with random sampling estimates in larger ranges, suggest that the true probabilities are considerably smaller than current upper bounds.
This is joint work with Carl Pomerance.

Neil Lyall, University of Georgia
Title: Embedding distance graphs into sets of positive density, I
Abstract: A distance graph is a graph $G = (V, E)$ whose vertex set $V = \{v_1, ..., v_n\}$ is contained in a Euclidean space. We say that G can be embedded in a set A if A contains a set $W = \{w_1, ..., w_n\}$ such that $|w_i - w_j| = |v_i - v_j|$ for all edges $(i, j) \in E$. We show that all large dilates of G can be embedded into any set A of positive upper density of the d-dimensional Euclidean space as long as $\deg(G) < d$. This latter condition means for any j the number of edges (i, j) for which $i < j$is less than d. We show that similar results are possible for subsets of the integer lattice in dimensions $d > C \deg(G)^2$.

Akos Magyar, University of Georgia
Title: Embedding distance graphs into sets of positive density, II
Abstract: A distance graph is a graph $G = (V, E)$ whose vertex set $V = \{v_1, ..., v_n\}$ is contained in a Euclidean space. We say that G can be embedded in a set A if A contains a set $W = \{w_1, ..., w_n\}$ such that $|w_i - w_j| = |v_i - v_j|$ for all edges $(i, j) \in E$. We show that all large dilates of G can be embedded into any set A of positive upper density of the d-dimensional Euclidean space as long as $\deg(G) < d$. This latter condition means for any j the number of edges (i, j) for which $i < j$is less than d. We show that similar results are possible for subsets of the integer lattice in dimensions $d > C \deg(G)^2$.

Michael Maltenfort, Northwestern University
Title: Characterizing additive systems
Abstract: An additive system is a collection of sets that gives a unique way to represent either all nonnegative integers, or all nonnegative integers up to some maximum. A structure theorem of de Bruijn gives a certain form for an additive system of infinite size. This form is not, in general, unique. We improve de Bruijn's theorem by finding a unique form for an additive system of arbitrary size. Our proof gives a concrete construction that allows us to test easily whether a collection of sets is an additive system. We also show how to determine most of the structure of an additive system if we are only given its union.

Azita Mayeli, Queensborough Community College (CUNY)
Title: Recent developments in Gabor orthogonal bases on finite vector spaces
Abstract: We study orthogonal Gabor bases in $\mathbb{Z}_p^d$, the d-dimensional vector space over the cyclic group of prime order. These are bases of the form $\{g(x-a)\chi(x \cdot b)\}$, $a \in A$, $b \in B$. The function g is called the window, A a set of translations and B a set of modulations. The investigation has analytic, number theoretic and combinatorial aspects. The analytic arguments comes in the form of Fourier analytic inequalities, the number theoretic aspects involve the analysis of exponential sums and the combinatorial arguments involve tiling and the study of directions sets.
This is a joint work with Alex Iosevich and Jonathan Pakianathan.

Nathan McNew, Towson State University
Title: Primitive and geometric-progression-free sets without large gaps
Abstract: We show how the probabilistic method can be used to construct primitive sets (sets of integers where no integer divides another) with relatively small gaps between consecutive terms, substantially smaller than is known to hold for the primes. We then show how the same techniques can be used to improve the bounds obtained by He for geometric-progression-free sets.

Steven J. Miller, Williams College
Title: Ramsey theory over number fields and quaternions
Abstract: In Ramsey theory one wishes to know how large a collection of objects can be while avoiding a particular substructure. A problem of recent interest has been to study how large subsets of the natural numbers can be while avoiding 3-term geometric progressions. Building on recent progress on this problem, we consider the analogous problem over quadratic number fields. We first construct high-density subsets of the algebraic integers of an imaginary quadratic number field that avoid 3-term geometric progressions. When unique factorization fails or over a real quadratic number field, we instead look at subsets of ideals of the ring of integers. Our approach is to construct the sets greedily, a generalization of the greedy set of rational integers considered by Rankin. We then describe the densities of these sets in terms of values of the Dedekind zeta function. If time permits we will discuss similar results in other settings (such as the quaternions).
This work is from REU projects supervised jointly with Nathan McNew.

Mel Nathanson, CUNY
Title: Problems in additive number theory and applications of commutative algebra
Abstract: This will be a survey of open problems in additive number theory, with examples of the application of classical theorems in commutative algebra to obtain additive results.

Hans Parshall, University of Georgia
Title: Spherical quadrilaterals over finite fields
Abstract: Graham conjectures that a finite Euclidean point set is Ramsey if and only if it is spherical, but we do not even know if this is true for a generic four point subset of a circle. We will discuss analogous problems in vector spaces over finite fields. In joint work with Neil Lyall and Ákos Magyar, we show that counting operators associated to spherical configurations are controlled by geometric uniformity norms. This approach allows us to establish a finite field analogue of the first open case of Graham's conjecture.

Giorgis Petridis, University of Georgia
Title: An inverse sum-product result in arbitrary fields
Abstract: One of the fundamental results in sum-product theory is, given two finite sets A, X in a field $\mathbb{F}$, to obtain a non-trivial lower bound for the cardinality of the set

$$A + XA = \{a + xb : a, b \in A, x \in X\}.$$

The lower bound $|A||X|^{1/2}$ is, for example, known when $\mathbb{F} = \mathbb{R}$ or $\mathbb{F} = \mathbb{F}_p$, a prime order finite field (mild conditions on $|X|, |A|$ are necessary).

The presence of subfields causes problems in positive characteristic. Take for example X to be a subset of a proper (and finite) subfield and A to be a vector space over the subfield. Then $A + XA = A + A = A$. Similarly, if A is a subset of the vector space with relative density $1/K$, then $|A + XA| \leq K|A|$.

In joint work with Brendan Murphy we established an inverse result by showing that these are the only examples where $|A + XA|$ can be nearly minimum. More specifically, we proved that if $|A + XA| \leq K|A|$, then X is a subset of a (finite) proper subfield and A has "large" relative density in the vector space over the subfield it generates. Large here means a small integral power of K^{-1}, which is close to being optimal.

We will discuss the relation to Bourgain's powerful bounds on the number of solutions to the equation

$$a_1 - a_2 = x(a_3 - a_4)\ a_i \in A,\ x \in X$$

and on products of differences in arbitrary finite fields.

Mengquing Qin, Missouri State University
Title: Bounds on $AA + AA$ in terms of AA in finite fields
Abstract: We present some results relating the relative sizes of AA and $AA + AA$, where A is a subset of a finite field. These follow as consequences of results due to Hart, Iosevich, and Solymosi. We also provide some simple corollaries and applications.

Sinai Robins,

Title: Discrete volumes and Fourier transforms of polytopes, and solid angle sums

Abstract. A discretized version of the volume of a polytope may be given by the number of integer points in P. There are infinitely many different families of discretized volumes of polytopes, but a particularly elegant one is the solid angle sum of a polytope, defined as follows. Place a very small sphere, centered at each lattice point of a given lattice $L \subseteq \mathbf{R}^d$ and consider the proportion of that sphere that intersects the polytope P, called a local solid angle. Now translate this small sphere to an arbitrary lattice point of L, and again consider the local solid angle contribution, relative to P. If we sum all of these contributions, over all lattice points of L, we get the solid angle sum of P, a nice discrete measure of the volume of P. It turns out that if we dilate P by an integer t (called the dilated polytope tP) and assume that the vertices of P lie on the lattice L, then this solid angle sum is a polynomial in the positive integer parameter t, and this polynomial is traditionally called AP(t) and first studied in detail by I. G. Macdonald. The coefficients of this polynomial encode certain geometric and number-theoretic properties of the polytope P, but they are still mysterious and not easy to compute in general. Here we extend the theory of these solid angle sums to all positive real dilations t, for any real polytope (so that its vertices do not necessarily lie on the lattice L). One of our ingredients is a detailed description of the Fourier transform of the polytope P. This transform method uses the formula of Stokes, which is a way of integrating by parts in several variables. Another key tool for us is the Poisson summation formula, applied to smoothings of the indicator function of the polytope P. It turns out that the combinatorics of the face poset of P plays a central role in the description of the Fourier transform of P, and in keeping track of the infinite series that pop out of Poisson summation. We also obtain a surprisingly simple closed form for the codimension-1 coefficient of the solid angle polynomial AP (t), for any real dilate of a rational polytope, extending previously known results about this codimension-1 coefficient. This is joint work with Ricardo Diaz and Quang-Nhat Le.

Ryan Ronan, CUNY Graduate Center

Title: An asymptotic for the growth of Markoff-Hurwitz tuples satisfying a congruence relation

Abstract: Consider the Markoff-Hurwitz equation

$$x_1^2 + x_2^2 + \cdots + x_n^2 = ax_1x_2\cdots x_n$$

for integer parameters $n \geq 3$, $a \geq 1$. In this talk, we establish an asymptotic count for the number of integral solutions with $\max\{x_1, x_2, \ldots, x_n\} \leq R$ and lying in a given congruence class modulo a square-free number q for which a particular transitivity property holds. When $n = 3$, $a = 1$ (or, equivalently, $a = 3$) this equation is known simply as the Markoff equation, and a number which appears as a solution to the Markoff equation is referred to as a Markoff number. Bourgain, Gamburd, and Sarnak recently showed that almost all Markoff numbers are composite. A crucial ingredient in their proof is a result due to Mirzakhani which provides an asymptotic formula for the number of Markoff triples below a given height and lying in a given congruence class. We use methods from symbolic dynamics to obtain the analogous count in the general case of arbitrary n and a.

This is joint work with Alex Gamburd and Michael Magee.

Csaba Sándor, Budapest University of Technology and Economics, Hungary
Title: A lower bound of Ruzsa's number related to the Erdős-Turán conjecture
Abstract: For a set $A \subseteq \mathbb{N}$ and $n \in \mathbb{N}$, let $R_A(n)$ denote the number of ordered pairs $(a, a') \in A \times A$ such that $a + a' = n$. The celebrated Erdős-Turán conjecture says that, if $R_A(n) \geq 1$ for all sufficiently large integers n, then the representation function $R_A(n)$ cannot be bounded. For any positive integer m, Ruzsa's number R_m is defined to be the least positive integer r such that there exists a set $A \subseteq \mathbb{Z}_m$ with $1 \leq R_A(n) \leq r$ for all $n \in \mathbb{Z}_m$. In 2008, Chen proved that $R_m \leq 288$ for all positive integers m. Together with Quan-Hui Yang we proved that $R_m \geq 6$ for all integers $m \geq 36$. This lower bound is nearly the best possible. We discuss related problems as well.

James Sellers, Pennsylvania State University
Title: An infinite family of congruences for ℓ-regular overpartitions
Abstract: We consider new properties of the combinatorial objects known as overpartitions (which are natural generalizations of integer partitions). In particular, we establish an infinite set of Ramanujan-type congruences for the restricted overpartitions known as ℓ-regular overpartitions. This significantly extends the recent work of Shen which focused solely on 3regular overpartitions and 4regular overpartitions.
This is joint work with Abdulaziz M. Alanazi and Augustine O. Munagi of University of the Witwatersrand, South Africa.

Steve Senger, Missouri State University
Title: Rainbow triangles
Abstract: We prove that, under some mild conditions, any coloring of two-dimensional vector space over a finite field satisfying some maximum color class constraints must have a unit equilateral triangle with each vertex from a different color class. We also discuss some related results.

Satyanand Singh, New York City Tech (CUNY)
Title: A sinusoidal twist with exponential influences
Abstract: The under damped oscillating spring mass dynamical system with displacement shaped like $u(t) = Pe^{-\gamma t/2m} \cos(\mu t - \delta)$ and no external force is most interesting as a discrete mathematical model with applications in quantum dynamics and cryptography. If we let m be the mass, γ the damping constant, c the spring constant and $t = \tau_1 > 0$ the instant where the first extreme value occurs, we will show that the distance S traveled by a particle modeled in this system results in the unexpectedly nice closed form expression given below.

$$S = P\Big[|\cos(\phi)|\,(1 + \coth(\gamma\pi/4m\mu))\,e^{-\gamma\tau_1/2m} - |\cos(\delta)|\,\mathrm{sgn}\,(u(0)u(\tau_1))\Big].$$

Jonathan Sondow, New York
Title: Power-sum denominators
Abstract: The *power sum* $1^n + 2^n + \cdots + x^n$ has been of interest to mathematicians since classical times. Johann Faulhaber, Jacob Bernoulli, and others who followed expressed power sums as polynomials in x of degree $n+1$ with rational coefficients. Here we consider the denominators of these polynomials, and prove some of their properties. A remarkable one is that such a denominator equals $n+1$ times the squarefree product of certain primes p obeying the condition that the sum of the base-p digits of $n+1$ is at least p. As an application, we derive a squarefree product formula for the denominators of the Bernoulli polynomials.
This is joint work with Bernd C. Kellner. Our paper will appear in the *American Mathematical Monthly* in 2017/18.

Jack Sonn, Technion, Israel
Title: Quadratic residues and difference sets
Abstract: It has been conjectured by Sárközy that with finitely many exceptions, the set of quadratic residues modulo a prime p cannot be represented as a sumset $\{a + b \colon a \in A, b \in B\}$ with non-singleton $A, B \subseteq \mathbf{F}_p$. The case $A = B$ of this conjecture has been recently established by Shkredov. The analogous problem for differences remains open: is it true that for all sufficiently large primes p, the set of quadratic residues modulo p is not of the form $\{a' - a'' \colon a', a'' \in A,\ a' \neq a''\}$ with $A \subseteq \mathbf{F}_p$?

We attack here a presumably more tractable variant of this problem, which is to show that there is no $A \subseteq \mathbf{F}_p$ such that every quadratic residue has a *unique* representation as $a' - a''$ with $a', a'' \in A$, and no non-residue is represented in this form. We have produced a number of necessary conditions for the existence of such A, involving for the most part the behavior of primes dividing $p - 1$. These conditions enable us to rule out all primes p in the range $13 < p < 10^{20}$ (the primes $p = 5$ and $p = 13$ being conjecturally the only exceptions). The talk will focus on one of these conditions, which emerges from an algebraic number theoretic approach to the problem.
This is joint work with Vsevolod Lev.

Yoni Stancescu, Afeka College, Israel
Title: Selected problems and results in inverse additive number theory
Abstract: We shaill discuss a number of problems and results in combinatorial number theory and we will mostly focus on multi-dimensional sets in torsion free groups.

Stefan Steinerberger, Yale University
Title: Strange new interactions between analysis and number theory
Abstract: I will discuss three surprising interactions between analysis and number theory: (1) a result about maximal, local averages of functions that seems to need the Lindemann-Weierstrass theorem, (2) a curious sum of sines that knows the difference between rational and irrational numbers, and (3) mysterious patterns in an old 1962 integer sequence of Stanislaw Ulam ($300 prize for an explanation).

Salvatore Tringali, Institute for Mathematics and Scientific Computing, University of Graz, Austria
Title: On the Interplay between arithmetic combinatorics and factorization theory
Abstract: From a classical point of view, factorization theory is all about various phenomena arising from the non-uniqueness of factorization in atomic monoids (and rings), and their classification by an assortment of algebraic, arithmetic, or combinatorial invariants. The subject has become more and more popular since the publication of Geroldinger and Halter-Koch's 2006 monograph, which is entirely devoted to the commutative and cancellative case.

The main goal of the talk is to provide some evidence in support of the idea that arithmetic combinatorics and factorization theory can draw great benefit from the interaction with each other. More specifically, we will present an overview of a couple of recent developments in this direction, which have also resulted into a generalization of fundamental aspects of factorization theory to non-cancellative or non-commutative settings:

(1) Power monoids and restricted power monoids, whose arithmetic is, in some relevant cases (namely, for Dedekind-finite, aperiodic monoids), naturally linked to various properties of non-empty finite subsets of $\mathbf{N}$ that can or cannot be written as a sumset of a prescribed number of irreducible (or primitive) sets;
(2) The structure theorem for directed families of sets of non-negative integers, along with applications to the unions of sets of lengths of unit-cancellative atomic monoids.

This is joint work with Fan (arXiv:1701.09152) and Fan, Geroldinger, and Kainrath (arxiv:1612.03116).

Yuri Tschinkel, Courant Institute, NYU
Title: Rationality problems
Abstract: I will discuss recent advances in algebraic geometry, inspired by number theory. This is joint work with B. Hassett and A. Pirutka.

Ajmain Yamin, Bronx High School of Science
Title: "An n-dimensional Calkin-Wilf tree and its graph structure"
Abstract: The Calkin-Wilf tree is an infinite binary tree in which the root is labeled $1/1$ and the children of an arbitrary node a/b are $a/(a+b), (a+b)/b$. A positive n-dimensional fraction is a formal symbol $a_1/a_2/.../a_n/b$, where $a_i, b \in \mathbf{Z}^+$ and is in reduced form if and only if $\gcd(a_1, a_2, ..., a_n, b) = 1$. The n-dimensional Calkin-Wilf tree is a generalization of the Calkin-Wilf tree, where the Calkin-Wilf tree is the 1-dimensional case. The n-CW tree has root $1/1/\cdots/1/1$ and the children of an arbitrary node $a_1/a_2/.../a_n/b$ are all the n-dimensional fractions $x_1/x_2/\cdots/x_n/y$ which satisfy $(a_1, a_2, \cdots, a_n, b)$ replaces $(x_1, x_2, \cdots, x_n, y)$ under one iteration of the Euclidean Algorithm. It follows immediately that each reduced positive n-dimensional fraction labels exactly once. We study the nontrivial graph structure that emerges from this definition. A notion of associated partition is crucial. First, we obtain an expression for the out-degree of an arbitrary node which only depends on the associated partition. Next, we make a statement about which associated partitions are allowed in the child-set. Finally, we construct a finite directed graph,

Diagram$(n+1)$, of associated partitions which gives the complete graph structure of the n-dimensional Calkin-Wilf tree.

Yifan Zhang, Central Michigan University
Title: A combinatorial proof for the generating function of the product of second-order recurrence sequences
Abstract: In this paper, we establish a combinatorial method to obtain the generating function for the product of finitely many second-order recurrence sequences. We give several examples including the generating function for the power of binomial coefficients and for the product of Fibonacci and Lucas numbers.
This is joint work with George Grossman.