CANT 2016 Abstracts

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Combinatorial and Additive Number Theory

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Sarfraz Ahmad, Comsats Institute of Information Technology, Lahore, Pakistan
Title: On the chain blockers of the poset $C_5 \times C_b$
Abstract: Let $P = C_a \times C_b$ be a poset where C_i is the i-element chain $1 < \cdots < i$ for $i \geq 1$. A chain blocker for P is defined as a subset $A \subseteq P$ such that A is inclusionwise minimal with the property that every maximal chain in P contains at least one element of A. We discuss new combinatorial interpretation of the convoluted Catalan numbers $C(n, k) := \frac{k}{2n-k}\binom{2n-k}{n}$ and Catalan numbers are introduced in term of the chain blockers.

We are interested in chains blockers as they appear for a poset P as the minimal nonfaces of simplicial complex Δ_P^* which is Alexander dual to the simplicial complex Δ_P whose minimal non-faces are the maximal chains in P. Recall that a simplicial complex Δ over a ground set Ω is a subset $\Delta \subseteq 2^\Omega$ such that $A \subseteq B \in \Delta$ implies $A \in \Delta$. In this talk we will give a complete description of the chain blockers of $C_5 \times C_b$ for $b \geq 1$.

Paul Baginski, Fairfield University
Title: Arithmetic progressions, nonunique factorization, and additive combinatorics in the group of units mod n
Abstract: For integers $0 < a \leq b$, the arithmetic progression $M_{a,b} := a + b\mathbb{N}$ is closed under multiplication if and only if $a^2 \equiv a \mod b$. Any such multiplicatively closed arithmetic progression is called an arithmetic congruence monoid (ACM). Though these $M_{a,b}$ are multiplicative submonoids of $\mathbb{N}$, their factorization properties differ greatly from the unique factorization one enjoys in $\mathbb{N}$.

In this talk we will explore the known factorization properties of these monoids. When $a = 1$, these monoids are Krull and behave similarly to algebraic number rings, in that they have a class group which controls all the factorization. Combinatorially, factorization properties correspond to zero-sum sequences in the group. However, when $a > 1$, these monoids are not Krull and thus do not have a class group which fully captures the factorization behavior. Nonetheless, an ACM can be associated to a finite abelian group, whose additive combinatorics relate to the factorization properties of the ACM. We will pay particular attention to the factorization property of elasticity and its connection to sequences in the group which attain certain sums while avoiding others.

Gautami Bhowmik, Universit of Lille, France
Title: A combinatorial problem from a mediaeval Sanskrit text
Abstract: We will explain the construction and enumeration of pan-diagonal magic squares as they appear in *Ganitakaumudi* (1356).

Pierre Bienvenu, University of Bristol, UK
Title: Higher dimensional Siegel-Walfisz theorem
Abstract: The Green-Tao theorem provides asymptotics for the number of prime tuples of the form $(\psi_1(n), \ldots, \psi_t(n))$ when n ranges among the integer vectors of a convex body $K \subset [-N, N]^d$ and $\Psi = (\psi_1, \ldots, \psi_t)$ is a system of affine linear forms whose linear coefficients remain bounded (in terms of N). In the $t = 1$ case, the Siegel-Walfisz theorem shows that the asymptotic still holds when the coefficients vary like a power of $\log N$. We prove a higher dimensional (i.e. $t > 1$) version of this fact. To do so, we basically rely on the work on Green and Tao, but inject new ideas to overcome the substantial use they make of the boundedness of the coefficients.

Arnab Bose, University of Lethbridge, Canada
Title: Investigations on some exponential congruences
Abstract: Around 1981, Selfridge asked for what positive integers a and b with $a > b$, does $2^a - 2^b$ divide $n^a - n^b$ for all $n \in \mathbb{N}$. The problem was independently solved by various people in different contexts, notably C. Pomerance (1977), Sun Qi and Zhang Ming Zhi (1985). In this talk, we study their ideas and prove a generalization of the problem, in the elementary number theoretic sense and also in algebraic number fields. Further, we develop ideas to give a conditional resolution and generalizations to another problem by H. Ruderman which is closely related to Selfridge's problem. (Joint work with Amir Akbary)

Kamil Bulinski, University of Sydney, Australia
Title: Twisted Multiple Recurrence and patterns in large subsets of $\mathbb{Z}^d$
Abstract: Recurrence is a fundamental notion in Ergodic Theory. It has a strong relationship with combinatorial number theory, as was first shown by Furstenberg when he proved a multiple recurrence result and deduced from it Szemerédi's theorem on arithmetic progressions. After a brief review of the relationship between recurrence and the existence of configurations in dense sets, I will present some new "twisted" multiple recurrence results obtained with M.Björklund (Chalmers) and their applications to finding "twisted" configurations in positive density subsets of $\mathbb{Z}^d$. This has many corollaries on the structure of images of positive density subsets of $\mathbb{Z}^d$ under quadratic forms, as well as other homogeneous polynomials with a large group of symmetries.

Sam Cole, University of Illinois at Chicago
Title: Planted partitions in random graphs
Abstract: In the planted partition problem, $n = ks$ vertices of a random graph are partitioned into k unknown "clusters," each of size s. Edges between vertices in the same cluster and different clusters are included with constant probability p and q, respectively (where $0 \leq q < p \leq 1$). The goal is to recover the unknown clusters from the randomly generated graph. I will give a brief survey of results for this problem and present a simple spectral algorithm that, with high probability, recovers the partition as long as the cluster sizes are at least $\Omega(\sqrt{n})$.

Colin Defant, University of Florida
Title: The unitary Cayley graph of $\mathbb{Z}/n\mathbb{Z}$
Abstract: If R is a commutative ring with 1, then the *unitary Cayley graph of* R, denoted G_R, is the graph with vertex set $V(G_R) = R$ and edge set

$$E(G_R) = \{\{x, y\} : x - y \in R^\times\}.$$

In particular, if $x, y \in \mathbb{Z}/n\mathbb{Z}$, then x and y are adjacent in $G_{\mathbb{Z}/n\mathbb{Z}}$ if and only if $x - y$ is relatively prime to n. In this talk, we give a simple formula for the number of cliques of order m in $G_{\mathbb{Z}/n\mathbb{Z}}$ for any positive integers m and n. This formula involves a class of arithmetic functions, known as Schemmel totient functions, that generalize the Euler totient function. We also discuss attempts to determine the domination numbers of the graphs $\mathbb{Z}/n\mathbb{Z}$. This problem is related to the Jacobsthal function j.

Mohamed El Bachraoui, United Arab Emirates University
Title: On the polynomiality of $\frac{1-q^b}{1-q^a}\begin{bmatrix} n \\ m \end{bmatrix}$
Abstract: In this talk we will give sufficient conditions for $\frac{1-q^b}{1-q^a}\begin{bmatrix} n \\ m \end{bmatrix}$ to be a polynomial with nonnegative integer coefficients. This unifies a variety of special cases already known in literature.

George Grossman, Central Michigan University
Title: Combinatorial identities with Fibonacci and Lucas numbers
Abstract: In this talk we explore aspects of combinatorial identities. In particular, it is shown that each Fibonacci number, up to sign change, can be represented countably many distinct ways as sum of binomial coefficients.

Sandie Han, New York City Tech (CUNY)
Title: Characterization of matrix $\in GL_2(\mathbb{N}_0)$ as product of certain pairs of 2×2 matrices
Abstract: This is continuation of the study on the characterization of the positive linear fractional transformation (PLFT) of the form $\dfrac{az+b}{cz+d}$ in the generalized Calkin-Wilf tree, where a, b, c, d are non-negative integers and $ad - bc \neq 0$. The associated matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ can be represented as product of $A(v) = \begin{bmatrix} 1 & v \\ 0 & 1 \end{bmatrix}$, and $B(u) = \begin{bmatrix} 1 & 0 \\ u & 1 \end{bmatrix}$, for some integers $u, v \geq 1$.
Joint work with Masuda, Singh, and Thiel.

Brian Hopkins, St. Peter's University
Title: Color restricted n-color compositions
Abstract: Following work of Agarwal and Andrews on the analogous partitions, in 2000 Agarwal defined n-color compositions, where a part k can have one of k different colors. The dozen or so subsequent papers on this topic have primarily focused on the n-color compositions that arise when only certain parts are allowed. Here, we focus instead on restricting colors. The resulting sets of n-color compositions provide combinatorial interpretations for many known integer sequences. We have developed recurrence relations for three very general families of color restricted n-color compositions and, for several particular cases, direct formulas and bijections using the idea of spotted compositions introduced by the author in 2013.
Joint work with Hua Wang of Georgia Southern University.

Alex Iosevich, University of Rochester
Title: On analytic, combinatorial and number theoretic aspects of the distance graph
Abstract: Let R be an abelian ring, finite or infinite, and let $E \subset R^d$, a d-dimensional module over R. Given a non-zero $t \in R$, define the distance graph $G_t(E)$ by taking elements of E to be the vertices, and connecting two vertices $x, y \in R^d$ by an edge if $(x_1 - y_1)^2 + \cdots + (x_d - y_d)^2 = t$. We shall examine some basic properties of this graph when $R = \mathbb{R}$, $\mathbb{Z}$ and $\mathbb{Z}_p$. A variety of diverse and interesting mathematics quickly comes to the surface in response to even the most basic questions pertaining to this object.

William J. Keith, Michigan Technological University
Title: Partitions simultaneously regular, distinct, and/or flat
Abstract: Partitions with parts not divisible by m, appearing less than m times, or with first differences less than m (and first part less than m) form three equinumerous classes, mapped by Glaisher's map and conjugation. These classes have been part of the study of partitions since Euler's initial work in the area. Yet few if any results have been found for partitions fixed by these maps, which satisfy two or three of these conditions, except for the classical result that partitions into distinct odd parts are in bijection with the self-conjugate partitions. In this talk we explore some of their properties and produce some results and questions.

Mizan Khan, Eastern Connecticut State University
Title: On White's characterization of empty lattice tetrahedra
Abstract: In 1963 White gave an elegant characterization of lattice tetrahedra that did not contain any lattice points other than their vertices. We will discuss a proof of White's theorem due to Karen Rogers.

S. V. Konyagin, Steklov Mathematical Institute of the Russian Academy of Sciences
Title: Number of nontrivial solutions of an equation with reciprocals
Abstract: I will discuss our joint results with M. A. Korolev on an upper estimate for the number of nontrivial solutions to the equation

$$\frac{1}{x_1} + \cdots + \frac{1}{x_r} = \frac{1}{y_1} + \cdots + \frac{1}{y_r}$$

where $x_1, \ldots, x_r, y_1, \ldots, y_r$ are positive integers not exceeding N. A solution is said to be nontrivial if $\{y_1, \ldots, y_r\}$ is not a permutation of $\{x_1, \ldots, x_r\}$.

Ben Krause, University of British Columbia
Title: Discrete analogues in harmonic analysis: A monomial Carlson theorem
Abstract: We give sufficient conditions on Λ under which the discrete maximal functions below are bounded on $\ell^2(\mathbb{Z})$.

$$\mathcal{C}_{d,\Lambda} f(n) := \sup_{\lambda \in \Lambda} \left| \sum_{m \neq 0} f(n-m) \frac{e^{2\pi i \lambda m^d}}{m} \right|, \ d \geq 2.$$

The set Λ can be certain kinds of Cantor sets, for instance. The integral version of this result holds with no restriction on Λ, and is due to Eli Stein and Stephen Wainger.
Joint work with Michael Lacey.

Urban Larsson, Dalhousie University, Halifax, Canada
Title: Sumsets and impartial games
Abstract: This talk concerns the star-operator of impartial vector subtraction games. We show that reflexive (limit) games satisfy certain properties of sumsets, depending on the convention being either normal- or misere play.
Joint work with Silvia Heubach and Matthieu Dufour.

Jiange Li, University of Delaware
Title: An entropy analogue of the more sums than differences problem
Abstract: We study the comparison of Shannon entropies of sum and difference of i.i.d. random variables. We show that they can differ by an real number additively, but not too much multiplicatively. The study is related to the more sums than differences problem in additive combinatorics.

Ray Li, Carnegie-Mellon, **Steven J. Miller**, Williams College, **Zhao Pan**, Carnegie-Mellon, and **Huanzhong Xu**, Carnegie-Mellon
Title: Convergence rates in generalized Zeckendorf decomposition problems
Abstract: We report on two projects on generalized Zeckendorf decompositions. The first concerns rates of convergence. Zeckendorf proved any integer can be decomposed uniquely as a sum of non-adjacent Fibonacci numbers, F_n. For Fibonacci numbers, the fraction of gaps that are size k, P_k $(k \geq 2)$, over all $m \in [F_n, F_{n+1})$ approaches $1/\phi^k$ as $n \to \infty$. Recently Bower et al. provided an explicit computation of P_k for all positive linear recurrence sequences $\{G_n\}_{n\in\mathbb{N}}$ from the recurrence relation. We extend these results and prove that, if $P_k(n)$ denotes the fraction of gaps that are size k for $m \in [G_n, G_{n+1})$, then $|1 - \frac{P_k(n)}{P_k}| = \frac{k+O(1)}{n+O(1)}$. Furthermore, we explicitly compute the $O(1)$ terms in the case of the Fibonacci numbers, and discuss its determination in many cases. We also report on some interesting numerical observations on approximating these gap probabilities, and discuss the determination of a transition window for the observed behavior.

The second project concerns positive linear recurrence sequence (PLRS), which are of the form $H_{n+1} = c_1 H_n + \cdots + c_L H_{n+1-L}$ with each $c_i \geq 0$ and $c_1 c_L > 0$, with appropriately chosen initial conditions. There is a notion of a legal decomposition generalizing the non-adjacency condition from the Fibonacci numbers (roughly, given a sum of terms in the sequence we cannot use the recurrence relation to reduce it) such that every positive integer has a unique legal decomposition of terms in the sequence. Previous work proved not only that a decomposition exists, but that the number of summands $K_n(m)$ in legal decompositions of $m \in [H_n, H_{n+1})$ converges to a Gaussian. Using partial fractions and generating functions it is easy to show the mean and variance grow linearly in n: $an+b$ and $cn+d$; the difficulty is proving a and c are positive. Currently the only way to do this requires delicate analysis of polynomials related to the generating functions and characteristic polynomials, and is algebraically cumbersome. We introduce new, elementary techniques that bypass these issues. The key insight is to use induction and bootstrap bounds through conditional probability expansions to show the variance is unbounded, and hence $c > 0$ (the mean is handled easily through a simple counting argument). These arguments can be generalized to other sequences that have a notion of legal decomposition and unique decomposition.

Neil Lyall, University of Georgia
Title: Some new results in geometric Ramsey theory
I plan to give a quick introduction to geometric (density) Ramsey theory, introduce a new perspective on the known results in the field (which lead to simplified proofs), and ultimately discuss new results, in particular, the fact that (geometric) squares are density Ramsey.

Akos Magyar, University of Georgia
Title: A Roth type theorem for dense subsets of $\mathbf{R}^d$
Abstract: It is shown that dense subsets of $\mathbf{R}^d$ contain 3-term progressions of all sufficiently large gaps, when the gap size is measured in the ℓ_p-metric for $p > 1$ and p not equal to 2. This is known to be false in the Euclidean ℓ_2-metric as well as in the ℓ_1-metric, and one of our goal is to understand this phenomenon.
Joint work with B. Cook and M. Pramanik.

Ariane Masuda, New York City Tech (CUNY)
Title: Weierstrass semigroups on Kummer extensions
Abstract: In this talk we will show how one can compute the Weierstrass semigroup at one totally ramified place and at two certain totally ramified places for a Kummer extension. We will also discuss some examples of one- and two-point Goppa codes with good parameters derived from our results.
Joint work with Luciane Quoos and Alonso Sepúlveda.

Nathan McNew, Towson University
Title: Counting Integers divisible by a large shifted prime
Abstract: In 1980 Erdos and Wagstaff showed that most positive integers are not divisible by any “large” shifted primes. We improve upon this result by obtaining precise estimates for the count of integers up to x divisible a shifted prime $p-1 > y$ in essentially the full range of x and y. In particular, we show that as x and y tend to infinity this count is at most $x/(\log y)^{b+o(1)}$, where $b = 1 - (1 + \log\log 2)/\log 2$ is the Erdős-Ford-Tenenbaum constant, and that this bound is optimal for a large range of y.

Steven J. Miller, Williams College, **Victor Xu**, Carnegie-Mellon, and **Xiaorong Zhang**, Carnegie-Mellon
Title: Existence of MSTD subsets and divots in the missing sums distribution
Abstract: We report on two projects on More Sum Than Difference (MSTD) sets. For the first, let S be a finite subset of $\mathbb{Z}$, and define the sumset $S+S = \{s+s' : s, s' \in S\}$ and the difference set $S-S = \{s-s' : s, s' \in S\}$; S is called an MSTD set if $|S+S| > |S-S|$. We investigate the existence of MSTD sets in subsets of $\mathbb{Z}$. We first prove that the Fibonacci sequence contains no MSTD subsets. Then we derive a sufficient condition for an integer sequence to have only finitely many MSTD subsets based on the growth rate of the sequence. Using a standard number theory conjecture, we are able to show that the set of prime numbers contains infinitely many MSTD sets.

For the second, we choose A uniformly at random from subsets of $\{0, \ldots, n-1\}$, let $A+A = \{x+y : x, y \in A\}$, and let $m_n(A)$ denote the number of missing sums in $A+A$ (i.e., $2n-1-|A+A|$). Previous work showed that as $n \to \infty$ the distribution of $m_n(A)$ has a divot at 7: $m(7) < m(6) < m(8)$ (where $m(k)$ denotes the limiting probability); that is, a random subset A is more likely to have its sumset $A+A$ missing 6 or 8 elements than missing 7 elements. We investigate what happens under different models of choosing A; each element of $\{0, \ldots, n-1\}$ is independently chosen to be in A with probability p (our initial model is equivalent to $p = 1/2$). We show that if $p > .966$ then as $n \to \infty$ there is a divot at 1. Further research is ongoing to determine the movement of the divot as the probability of selecting elements changes, and if and when there are multiple divots for certain probabilities p.

Amanda Montejano, UMDI-Facultad de Ciencias, Universidad Nacional Autónoma de México, Querétaro, México
Title: Zero-sum k-blocks in $\{-1, +1\}$ bounded sum sequences
Abstract: In this talk, we consider problems and results that go in the opposite direction of the classical theorems in the discrepancy theory. The following statement gives a flavor of our approach. Let t, k and q be integers such that $q \geq 0$, $0 \leq t < k$, and $t \equiv k \pmod 2$, and take $s \in [0, t+1]$ as the unique integer satisfying $s \equiv q + \frac{k-t-2}{2} \pmod{t+2}$. Then, for any integer

$$n \geq \frac{1}{2(t+2)}k^2 + \frac{q-s}{t+2}k - \frac{t}{2} + s$$

and any function $f : [n] \to \{-1, 1\}$ with $|\sum_{i=1}^{n} f(i)| \leq q$, there is a block of k consecutive terms (k-block) $B \subset [n]$ with $|\sum_{x \in B}^{n} f(x)| \leq t$. Moreover, this bound is sharp for all the parameters involved and a characterization of the extremal sequences is given. For example, taking $t = 0$, $k = 6$ and $q = 2$, we obtain that, if $n \geq 15$ then any function $f : [n] \to \{-1, 1\}$ with $|\sum_{i=1}^{n} f(i)| \leq 2$ contains a zero-sum 6-block, and this is not true for $n = 14$, where the only function (up to multiplication by -1) without zero-sum 6-blocks can be represented in the following way:

$$- - + + + + - - + + + + - -,$$

where the $+$'s and $-$'s represent the values $+1$ or, respectively, -1 at the corresponding position. This is a joint work with Yair Caro and Adriana Hansberg.

Brendan Murphy, University of Rochester
Title: Half-way to Szemeredi-Trotter over finite fields
Abstract: Recently, Roche-Newton, Rudnev, and Shkredov proved strong sum-product estimates over finite fields using Rudnev's point-plane incidence bound. We present a generalization of their method based on a bound for the number of "collisions of images of lines." This bound allows us to prove that the point set $P = A \times A$ determines at most $O(|A|^{9/2})$ collinear triples, which is halfway between the trivial bound of $O(|A|^5)$ and the sharp bound of $O(|A|^4 \log |A|)$ provided by the Szemeredi-Trotter theorem, valid over $\mathbb{R}$ and $\mathbb{C}$.
Joint work with Esen Aksoy-Yazici, Misha Rudnev, and Ilya Shkredov.

Rishi Nath, York College (CUNY)
Title: Sequences of simultaneous core partitions
Abstract: A simultaneous (s,t)-core partition is one in which neither s nor t appears in the set of associated hook numbers. We discuss some new results on simultaneous $(s, ms\pm1)$-cores and $(2k-1, 2k+1)$-cores and discuss some enumerative coincidences for which mappings have yet to be found.

Mel Nathanson, Lehman College (CUNY)
Title: Comparison estimates for linear forms
Abstract: Let M be an R-module. Consider the h-ary linear form $\Phi : M^h \to M$ defined by $\Phi(t_1, \ldots, t_h) = \sum_{i=1}^{h} v_i t_i$ For every subset A of M, define

$$\Phi(A) = \{\Phi(a_1, \ldots, a_h) : (a_1, \ldots, a_h) \in A^h\}.$$

with nonzero coefficient sequence $(v_1, \ldots, v_h) \in R^h$. For every nonempty subset I of $\{1, 2, \ldots, h\}$, define the *subsequence sum* $s_I = \sum_{i \in I} v_i$. Let $\mathcal{S}(\Phi) = \{s_I : I \subseteq \{1, 2, \ldots, h\}, I \neq \emptyset\}$ be the set of all nonempty subsequence sums of the sequence of coefficients of Φ. Let $R^\times$ be the group of units of the ring R.

Theorem. *Let Υ and Φ be linear forms with coefficients in the ring R. If $0, 1 \in \mathcal{S}(\Upsilon)$ and if $\mathcal{S}(\Phi) \subseteq R^\times$, then for every $\varepsilon > 0$ there exists a finite R-module M and a subset A of M such that*

$$\Upsilon(A) = M$$

and

$$|\Phi(A)| < \varepsilon |M|.$$

For example, the linear forms $\Upsilon(t_1, t_2) = t_1 - t_2$ and $\Phi(t_1, \ldots, t_h) = \sum_{i=1}^{h} t_i$ satisfy the Theorem with $R = \mathbf{Z}$ or $R = \mathbf{Z}/m\mathbf{Z}$, where $p > h$ for every prime divisor p of m.

Péter Pál Pach, Budapest University of Technology and Economics, Hungary
Title: Progression-free sets in $\mathbb{Z}_4^n$
Abstract: We show that if the subset $A \subseteq \mathbb{Z}_4^n$ is free of three-term arithmetic progressions, then $|A| < 2(\sqrt{n}+1)4^{\gamma n}$, with an absolute constant $\gamma \approx 0.926$. That is, progression-free sets in $\mathbb{Z}_4^n$ are exponentially small.
Joint work with Ernie Croot and Vsevolod F. Lev.

Giorgis Petridis, University of Rochester
Title: Products of differences in finite fields
Abstract: We present recent progress on a sum-product in finite fields question. The question is to consider a set A in the prime order finite field F_p and ask how large must its cardinality be so that the set of products of differences of elements of A

$$(A - A)(A - A) = (a - b)(c - d) : a, b, c, dinA$$

is large?

Bennet, Hart, Iosevich, Pakianathan, and Rudnev showed that $|A| > p^{2/3}$ implies $(A - A)(A - A)$contains half the elements of F_p. This result is typical of a long list of sum-product questions in F_p, where the condition $|A| > p^{2/3}$ implies that a set derived by a combination of sums and products of elements of A has at least $p/2$ elements.

The purpose of the talk is to explain how prove the existence of an absolute constant c¿0 such that $|A| > p^{2/3-c}$ implies that $(A - A)(A - A)$ has at least p/2 elements. To the best of our knowledge this is the first instance in the literature where the $p^{2/3}$ threshold is beaten for such a type of sum-product question.

One of the ingredients is a recent result of Aksoy-Yazici, Murphy, Rudnev, and Shkredov, which allows one to have a not tiny c.

Sinai Robins, ICERM at Brown University, and University of Sao Paulo, Brazil
Title: Discrete volumes of polytopes, Fourier transforms of polytopes, and solid angle sums
Abstract. A discretized version of the volume of a polytope may be given by the number of integer points in $\mathcal{P}$. There are infinitely many different families of discretized volumes of polytopes, but a particularly elegant one is the "solid angle sum" of a polytope, defined as follows. Place a very small sphere, centered at each lattice point of a given lattice $\mathcal{L} \subset \mathbb{R}^d$, and consider the proportion of that sphere that intersects the polytope $\mathcal{P}$, called a local solid angle. Now translate this small sphere to an arbitrary lattice point of $\mathcal{L}$, and again consider the local solid angle contribution, relative to $\mathcal{P}$. If we sum all of these contributions, over all lattice points of $\mathcal{L}$, we get the "solid angle sum" of $\mathcal{P}$, a nice discrete measure of the volume of $\mathcal{P}$. It turns out that if we dilate $\mathcal{P}$ by an integer t (called the dilated polytope $t\mathcal{P}$) and assume that the vertices of P lie on the lattice $\mathcal{L}$, then this solid angle sum is a polynomial in the positive integer parameter t, and this polynomial is traditionally called $A_{\mathcal{P}}(t)$ and first studied in detail by I. G. Macdonald. The coefficients of this polynomial encode certain geometric and number-theoretic properties of the polytope $\mathcal{P}$, but they are still mysterious and not easy to compute in general.

Here we extend the theory of these solid angle sums to all positive real dilations t, for any real polytope (so that its vertices do not necessarily lie on the lattice $\mathcal{L}$). One of our ingredients is a detailed description of the Fourier transform of the polytope $\mathcal{P}$. This transform method uses the formula of Stokes, which is a way of integrating by parts in several variables. Another key tool for us is the Poisson summation formula, applied to smoothings of the indicator function of the polytope $\mathcal{P}$. It turns out that the combinatorics of the face poset of $\mathcal{P}$ plays a central role in the description of the Fourier transform of $\mathcal{P}$, and in keeping track of the infinite series that pop out of Poisson summation. We also obtain a closed form for the codimension-1 coefficient of the solid angle polynomial $A_P(t)$, extending previously known results about this codimension-1 coefficient. This is joint work with Ricardo Diaz and Quang-Nhat Le.

Bradley Rodgers, University of Michigan
Title: Divisor sums in short intervals and counts of plane partitions
Abstract: In this talk we will discuss recent joint work with Jon Keating, Edva Roditty-Gershon, and Zeev Rudnick regarding the distribution of sums of k-fold divisor functions over a random short interval. We will talk about new conjectures for the variance of these sums, and discuss an analogous result in a function field setting which motivates the conjectures and which has a surprising connection to plane partitions.

Ryan Ronan, CUNY Graduate Center
Title: An asymptotic for the growth of Markoff-Hurwitz tuples
Abstract: Consider the Markoff-Hurwitz equation

$$x_1^2 + x_2^2 + \cdots + x_n^2 = ax_1x_2\cdots x_n + k$$

for integer parameters $n \geq 3$, $a \geq 1$, $k \geq 0$. In this talk, we establish an asymptotic count for the number of integral solutions with $\max\{x_1, x_2, \ldots, x_n\} \leq R$. When $n = 3$, $a = 1$ (or, equivalently, $a = 3$), and $k = 0$ this equation is known simply as the Markoff equation, for which the asymptotic count was studied in detail by Zagier in 1982. The previous best result for $n \geq 4$ is due to Baragar in 1998 who established an exponential rate of growth with exponent $\beta > 0$ which is not, in general, an integer. We use methods from symbolic dynamics to improve this asymptotic count, and which yield a new interpretation of this exponent β in terms of the unique parameter for which there exists a certain conformal measure on projective space.
Joint work with Alex Gamburd and Michael Magee.

Tom Sanders, Oxford University, UK
Title: Monochromatic sums and products
Abstract: We shall discuss work with Ben Green on finding monochromatic quadruples $(x, y, x + y, xy)$ in finite colourings of $\mathbf{F}_p$.

James Sellers, Pennsylvania State University
Title: Infinitely many congruences modulo 5 for 4-colored Frobenius partitions
Abstract: In his 1984 AMS Memoir, Andrews introduced the family of functions $c\phi_k(n)$, which denotes the number of generalized Frobenius partitions of n into k colors. Recently, Baruah and Sarmah, Lin, Sellers, and Xia established several Ramanujan-like congruences for $c\phi_4(n)$ relative to different moduli. In this paper, which is joint work with Michael D. Hirschhorn of UNSW, we employ classical results in q-series, the well-known theta functions of Ramanujan, and elementary generating function manipulations to prove a characterization of $c\phi_4(10n+1)$ modulo 5 which leads to an infinite set of Ramanujan-like congruences modulo 5 satisfied by $c\phi_4$. This work greatly extends the recent work of Xia on $c\phi_4$ modulo 5.

Steven Senger, Missouri State University
Title: A few sum-product type estimates
Abstract: We will look at sets of the type $AA + 1$ and $(A - A)(A - A)$, where A is a subset of either the real numbers or a finite field.

Satyanand Singh, New York City Tech (CUNY)
Title: The odd behavior of Nathanson's lambda sequences
Abstract: For $\mathcal{A}$ any finite set of positive integers greater than 1, and $a \in \mathcal{A}$ we define the set $A_a = \{\varepsilon_j(a) \cdot a^j : j = 0, 1, 2, ...\}$, where $\varepsilon_j(a) \in \{\ 0, \pm 1, \pm 2, ..., \pm\lfloor a/2 \rfloor\}$. Nathanson studied the properties of the related $\lambda_{\mathcal{A}}(h)$ sequences. In this setting positive integers are partitioned as sums of elements from the set $\mathcal{S}_{\mathcal{A}} = \bigcup_{a \in \mathcal{A}} A_a$. Nathanson posed the problem to compute $\lambda_{\mathcal{A}}(h)$, where $\lambda_{\mathcal{A}}(h)$ is defined as the smallest positive integer that can be represented as the sum of elements of $\mathcal{S}_{\mathcal{A}}$ with length h, but that cannot be represented as a sum with length less than h. In this presentation we will restrict our study to sets of the shape $\mathcal{A} = \{2, n\}$ and odd $n > 1$ and illustrate how to generate the values of $\lambda_{2,n}(h)$ for fixed $h \in \{1, 2, 3\}$.

Jonathan Sondow, New York
Title: Babbage's (non)-primality tests
Abstract: In 1819 Charles Babbage published his only paper in number theory. I will recall his non-primality and primality tests, involving congruences for certain binomial coefficients. I will mention the known counterexamples–Wolstenholme primes squared–to the converse of Babbage's non-primality test, and prove a partial converse. Finally, I will give a generalization of his primality test.

Yoni Stanchescu, Afeka College and the Open University of Israel
Title: Small sumsets in torsion-free groups
Abstract: This talk will review some results and problems concerning small sumsets in torsion-free groups. We will discuss recent advancements in different classes of abelian and non-abelian groups. We will describe the structure of the extremal sets and we are going to prove some precise structure theorems for sets of lattice points of small doubling.

Johann Thiel, New York City Tech (CUNY)
Title: The growth of coefficients in certain PLFT (u, v)-Calkin-Wilf trees
Abstract: A positive linear fractional transformation (PLFT) is a function of the form $f(z) = \frac{az+b}{cz+d}$ where a, b, c, and d are nonnegative integer coefficients with determinant $ad - bc \neq 0$. Nathanson defined a PLFT (u, v)-Calkin-Wilf tree, with u, v positive integers, as an infinite rooted binary tree where every vertex is labelled by a PLFT using a simple set of rules. If a vertex is labelled by the PLFT $f(z)$, then the left child of the vertex is labelled by $L_u(f(z))$ and the right child is labelled by $R_v(f(z))$ where $L_u(z) = \frac{z}{uz+1}$ and $R_v(f(z)) = z + v$. In this talk we study the size of the coefficients of PLFTs appearing in certain PLFT (u, v)-Calkin-Wilf trees. Joint work with Sandie Han, Ariane M. Masuda, and Satyanand Singh.

Andrew Treglown, University of Birmingham, UK
Title: Maximal sum-free subsets in the integers
Abstract: Cameron and Erdős asked whether the number of maximal sum-free sets in $\{1,\ldots,n\}$ is much smaller than the number of sum-free sets. In the same paper they gave a lower bound of $2^{\lfloor n/4 \rfloor}$ for the number of maximal sum-free sets. We prove the following: For each $1 \leq i \leq 4$, there is a constant C_i such that, given any $n \equiv i \mod 4$, the set $\{1,\ldots,n\}$ contains $(C_i + o(1))2^{n/4}$ maximal sum-free sets. Our proof makes use of container and removal lemmas of Green, a structural result of Deshouillers, Freiman, Sós and Temkin and a recent bound on the number of subsets of integers with small sumset by Green and Morris.
Joint work with Jozsef Balogh, Hong Liu and Maryam Sharifzadeh.

Yuri Tschinkel, NYU
Title: Counting integral points on orbits of algebraic groups
Abstract: I will describe recent results and conjectures concerning the distribution of integral points of bounded height on orbits of algebraic groups.
Joint work with A. Chambert-Loir.

Van Vu, Yale University
Title: Sum-free sets in groups
Abstract: We discuss several questions concerning sum-free sets in groups, raised by Erdős in his survey *"Extremal problems in number theory"* (Proceedings of the Symp. Pure Math. VIII AMS) published in 1965. Among other things, we give a characterization for large sets A in an abelian group G which do not contain a subset B of fixed size k such that the sum of any two different elements of B do not belong to A (in other words, B is sum-free, or sum-avoiding, with respect to A). Erdős, in the above mentioned survey, conjectured that if $|A|$ is sufficiently large compared to k, then A contains two elements that add up to zero. This is known to be true for $k \leq 3$. We give counterexamples for all $k \geq 4$. On the other hand, using the new characterization result, we are able to prove a positive result in the case when $|G|$ is not divisible by small primes.
Joint work with Terence Tao